Probabilistic Methods in Combinatorics

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Assignment 5

To solve for the Example class on 25th March. Submit the solution of Problem 1 by Sunday 23th March if you wish feedback on it.

The solution of each problem should be no longer than one page!

Problem 1. Prove that there is an absolute constant $c > 0$ with the following property. Let A be an n by n matrix with pairwise distinct entries. Then there is a permutation of the rows of A so that no column in the permuted matrix contains an increasing subsequence of length at least $c\sqrt{n}$.

Problem 2. Prove that every three-uniform hypergraph with n vertices and $m \geq n/3$ hyperedges contains an independent set (i.e. a set of vertices containing no hyperedges) of size at least

$$\frac{2n^{3/2}}{3\sqrt{3}\sqrt{m}}.$$

Problem 3. Prove that if there exists some $0 \leq p \leq 1$ such that

$$\binom{n}{t} p^{\binom{t}{2}} + \binom{n}{k} (1-p)^{\binom{k}{2}} \leq n/2,$$

then $R(t,k) \geq n/2$. Using this, show that the Ramsey number $R(4,k)$ satisfies

$$R(4,k) \geq \Omega((k/\ln k)^2).$$